B.Sc. Part - I

Langrange's Equation of motion for Blows

Let $\dot{x}_0$ and $\dot{x}_1$ denotes the values of $\dot{x}$ before and after the action of the blows. Since the virtual moment of the effective impulses $\Sigma m(\dot{x}_1 - \dot{x}_0)$ etc. is equal to the virtual moment of the impressed blows. we have, for a variation in θ only,

$$\Sigma m\left[(\dot{x}_1 - \dot{x}_0)\frac{dx}{d\theta} + (\dot{y}_1 - \dot{y}_0)\frac{dy}{d\theta} + (\dot{z}_1 - \dot{z}_0)\frac{dz}{d\theta}\right]\delta\theta$$

$$= \Sigma m\left[X_1\frac{dx}{d\theta} + Y_1\frac{dy}{d\theta} + Z_1\frac{dz}{d\theta}\right]\delta\theta$$

Let T_0 and T_1 be the values of T just before and just after the blows.

Then from Langrange's equation

$$\left(\frac{dT}{d\theta}\right)_0 = \Sigma m\left[\dot{x}\frac{d\dot{x}}{d\theta} + \dot{y}\frac{d\dot{y}}{d\theta} + \dot{z}\frac{d\dot{z}}{d\theta}\right]_0$$

$$= \Sigma m\left[\dot{x}\frac{dx}{d\theta} + \dot{y}\frac{dy}{d\theta} + \dot{z}\frac{dz}{d\theta}\right]$$

$$\left(\frac{dT}{d\theta}\right)_1 = \Sigma m\left[\dot{x}\frac{dx}{d\theta} + \dot{y}\frac{dy}{d\theta} + \dot{z}\frac{dz}{d\theta}\right]$$

Hence the left hand of (1) is

$$\left[\left(\frac{dT}{d\theta}\right)_1 - \left(\frac{dT}{d\theta}\right)_0\right]\delta\theta$$

Also the right hand of (1)

$$= \left[\frac{dV_1}{dx}\frac{dx}{d\theta} + \frac{dV_1}{dy}\frac{dy}{d\theta} + \frac{dV_1}{dz}\frac{dz}{d\theta}\right]\delta\theta = \frac{dV_1}{d\theta}\delta\theta$$

where the δV_1 is the virtual work of the blows.

Hence δV_1 be ~~the~~ expressed in the form

$$\delta V_1 = P\delta\theta + Q\delta\theta + \cdots\cdots\cdots$$

the equation (1) can be written in the form

$$\left(\frac{dT}{d\theta}\right)_1 - \left(\frac{dT}{d\theta}\right)_0 = P \qquad (2)$$

and Similarly for the other equations.

The equation (2) may be written by integrating eqn

$$\frac{d}{dT}\left(\frac{dT}{d\theta}\right) - \frac{dT}{d\theta} = \frac{dV}{d\theta}$$

between the limits 0 and T where T is the infinitesimal time during which the blows last.

The integral of $\frac{d}{dt}\left(\frac{dT}{d\theta}\right)$ is $\left[\frac{dT}{d\theta}\right]_0^T$,

ie $\left[\frac{dT}{d\theta}\right]_1 - \left[\frac{dT}{d\theta}\right]_0$

Since $\frac{dT}{d\theta}$ is finite, its integral during the small time T is ultimately zero.

The integral of $\frac{dV}{d\theta}$ is $\frac{dV_1}{d\theta}$

Now we consider on three equal uniform rod AB, BC, CD are freely joined at B and C and the ends of A and D are fastened to smooth fixed pivotes whose distance apart is equal to the length of either rod. The frame being at rest in the form of a square, a below J is given perpendicularly to AB at its middle point and in the plane of the square. Shew that the energy set up is $\frac{3J^2}{40m}$, where m is the mass of each rod. Find also the blows at the joints B and C.

When AB and CD, has tuned through an angle θ, the energy of either is $\frac{1}{2} m \frac{4a^2}{3} \dot{\theta}^2$ and that of BC, which remains parallel AD, is $\frac{1}{2} m \cdot \frac{4a^2}{3} \theta^2$, and

and that of BC, which remains parallel to AD,

is $\frac{1}{2} m (2a\dot{\theta})^2$

$$\therefore T = 2 \cdot \frac{1}{2} m \cdot \frac{4a^2}{3} \dot{\theta}^2 + \frac{1}{2} \cdot m 4a^2 \dot{\theta}^2 = \frac{10ma^2}{3} \cdot \dot{\theta}^2$$

$$\therefore \left(\frac{dT}{d\theta}\right)_1 = \frac{20ma^2}{3} \dot{\theta}, \quad \text{and} \quad \left(\frac{dT}{d\theta}\right)_0 = 0$$

Also $\quad \delta V_1 = J \cdot a \delta\theta.$

Hence we have $\frac{20ma^2}{3} \theta = J \cdot a$

ie $\dot{\theta} = \frac{3J}{20ma}$

$$\therefore \text{required energy} = \frac{10ma^2 \dot{\theta}^2}{3}$$

$$= \frac{3J^2}{40m}.$$

If Y and Y_1 be the blows at the joints B and C then, by taking moments about A and D for the rods AB and DC, we have

$$m \cdot \frac{4a^2}{3} \dot{\theta} = J \cdot a - Y \cdot 2a$$

and $\quad m \cdot \frac{4a^2}{3} \dot{\theta} = Y_1 \cdot 2a$

$$\therefore Y = \frac{2J}{5} \quad \text{and} \quad Y_1 = \frac{J}{10}$$

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